American Academy of Actuaries' C-3 Phase II RBC/AG-43 Work Group
(Academy VAREQ)

EQUITY RETURN CALIBRATION CRITERIA

PRINCIPLE-BASED VALUATION FOR VARIABLE ANNUITIES UNDER C-3 PHASE II RBC and AG 43

June 4, 2013

AMERICAN ACADEMY of ACTUARIES

The Academy VAREQ[1] adopted an objective, analytical approach to examining equity returns in light of the existing calibration criteria

- The primary objective was to assess whether (and if so, how) the addition of post-2003 data (i.e., inclusive of the recent financial crisis) would materially change the calibration criteria[2]:
  - This included a review of the calibration criteria for US equity returns in AG 43 and C-3 Phase II RBC (hereafter, referred to as the "calibration criteria")
  - We examined the "reasonableness and robustness" of the existing (2005)[3] requirements given perceived recent market volatility and "financial crisis returns" (i.e., post 2007)
- We did not develop "new" calibration criteria from scratch; rather, we used the SLV[5] model as a reference point, employing similar analytical methods as the original Academy LCAS[4] in 2005 (with the addition of post-2003 data)
- While the SLV model was not the sole determinant of the calibration criteria, it exhibits the desirable characteristics of a real-world equity return model and is the foundation of the pre-packaged scenarios; accordingly, we use the SLV extensively as a reference point in the analysis
- Our goal here is to analyze the data objectively and offer insights using analytical methods similar to prior LCAS work
- We believe the analysis herein will be useful in the broader evaluation of the overall effectiveness of AG 43 and C-3 Phase II RBC for the valuation of guaranteed benefits on variable annuities

1. American Academy of Actuaries' C-3 Phase II RBC/AG-43 Work Group.
2. The criteria that apply to the calibration of economic scenario generators (ESGs) that simulate future equity returns for the valuation of variable annuities with guaranteed benefits.
3. See "Recommended Approach for Setting Regulatory Risk-Based Capital Requirements for Variable Annuities and Similar Products", Academy LCAS, June 2005 (hereafter referred to as the "June 2005 LCAS Report").
4. Academy's Life Capital Adequacy Subcommittee.
5. SLV = Stochastic Log Volatility. See *C3 Phase II Risk-Based Capital for Variable Annuities: Pre-Packaged Scenarios*, Academy LCAS, March 2005.

Based on the current analysis, the addition of post-2003 data would result in only minor technical adjustments to the calibration criteria

- The SLV model and the pre-packaged scenarios:
  - Satisfy the calibration criteria and fit the historic monthly return data very well
  - Produce volatility paths that are almost indistinguishable from history (i.e., clustering, spikes, etc.)
  - Reflect the potential for large 1-year market declines (e.g., -43.3%, Feb 2008 – Feb 2009) with relative frequency consistent with history (see slides 20-25)
- Hence, calibrated models capture recently witnessed short-term volatility and adverse market performance

- Based on our analysis, the Academy VAREQ sees no material statistical evidence to update the equity return calibration criteria at this time

Existing (2005) Equity Return Calibration Criteria

Percentile	1 year	5 years	10 years	20 years
2.5%	0.78	0.72	0.79	
5.0%	0.84	0.81	0.94	1.51
10.0%	0.90	0.94	1.16	2.10
90.0%	1.28	2.17	3.63	9.02
95.0%	1.35	2.45	4.36	11.70
97.5%	1.42	2.72	5.12	

Implied (Updated) Calibration Criteria (1956-2012 data)

Percentile	1 year	5 years	10 years	20 years
2.5%	0.78	0.70	0.78	
5.0%	0.84	0.81	0.93	1.49
10.0%	0.90	0.94	1.15	2.07
90.0%	1.28	2.15	3.59	8.90
95.0%	1.35	2.44	4.32	11.54
97.5%	1.42	2.70	5.06	

We recommend that a process for analyzing, testing and updating the calibration criteria be discussed and agreed upon before isolated adjustments are made

1 Recap of Equity Calibration Criteria for Principle-based Valuation

In June 2005, the Academy's Life Capital Adequacy Subcommittee[1] issued a report[2] on setting risk-based capital (RBC) for variable annuities

- The report proposed setting regulatory capital requirements for market risk (C-3) associated with guarantees on variable annuities according to company-specific models (subject to guidance and constraints)
- In particular, market risk factors (e.g., equity returns and interest rates) are developed from real-world stochastic simulations over the life of the business
- The Academy LCAS did not and the Academy VAREQ does not support mandating specific investment return models and parameters. Rather, both groups supported/support the use of company models and assumptions to generate scenarios, subject to the criteria set out in the requirements.
- With reference to the following chart, we continue to believe the advantages of allowing company determined scenarios outweigh the advantages of mandating models/parameters

On the selection and use of company-specified models and parameters for economic scenario generation

Advantages of company-determined scenarios	Advantages of prescribed scenarios
• Fosters R&D on new ESG models and approaches, allowing for the use of the latest tools and techniques • Promotes the use of the model beyond regulatory purposes (i.e., can satisfy the "use test" for decision-making) • Avoids undue emphasis being placed on the "correctness" of a single ESG model or software engine	• Standardized approach allows regulators greater comparability across companies • Potentially easier implementation for insurers due to lack of choice (i.e., "plug-and-play") • Potentially avoids errors in implementation (e.g., random number generation) and application

1. Hereafter referred to as the "Academy LCAS".
2. See *Recommended Approach for Setting Regulatory Risk-Based Capital Requirements for Variable Annuities and Similar Products*, Academy LCAS, June 2005.

The calibration criteria were developed from statistical analysis, but not based directly on history or on any specific model

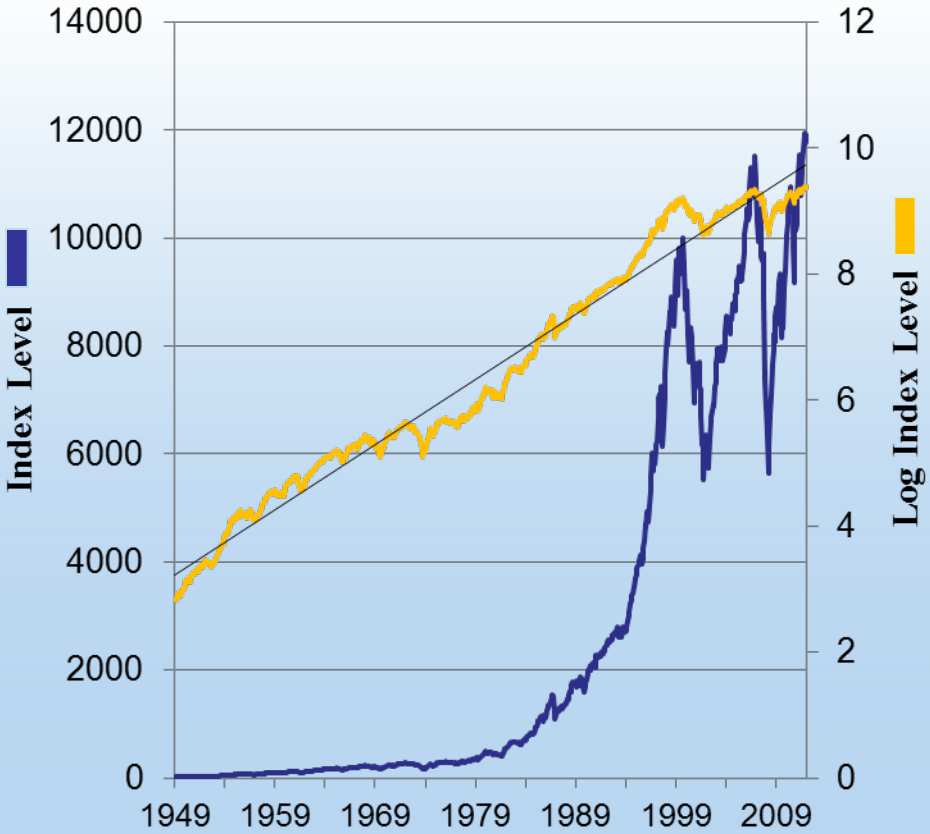
- A key component of the stochastic modeling of the future benefits/fees associated with variable annuities is the model used to simulate future investment return paths, particularly for equities.
- To narrow the range of practice to acceptable levels in the use of real world stochastic models for the equity returns, the Academy LCAS recommended so-called "equity return calibration criteria"
  - Expressed as "gross wealth ratios" (accumulated value of a unit) over various time horizons & percentiles; the same criteria were incorporated into AG 43
- The calibration criteria were based on extensive statistical analysis, but not directly derived from historic data. Rather, the criteria were developed from a variety of "suitable" stochastic models[1] fit to history.
- Key aspects under-pinning the design of the equity return calibration criteria (2005):
  - Used S&P500 daily total[2] return index data, 1955.12.31 to 2003.12.31 – *covered many market cycles*
  - The SLV[3] model was used as a reference due to its ability to capture "fat tails" and "negative skew," but it did not solely determine the criteria – *so as not to exclude other reasonably parameterized models*
  - A "long-run" average return of 8.75% p.a. imposed by the NAIC – *despite the data indicating higher average expected returns* (>10.5% over 1955-2003 and >9.6% over 1955-2012)
  - The criteria are "two-sided" (left and right tails) – *to reflect "call option-like" features[4] in VAs*
- As such, models that satisfy the criteria should produce scenarios more extreme than the historic experience, especially over longer timeframes (see slides 20-25)

1. Various versions of the SLV and Regime-switching lognormal model (RSLN) were considered due to their abilities to capture the "negative skew" and "fat tails" of history.
2. The total return index includes changes due to price movements (of the component stocks) and reinvestment of all dividends.
3. SLV = Stochastic Log Volatility. See *C3 Phase II Risk-Based Capital for Variable Annuities: Pre-Packaged Scenarios*, Academy LCAS, March 2005.
4. Call-option features include various forms of resets and ratchets of the guaranteed benefit levels. These benefits are more costly when markets rise (near term) and then fall.

The S&P500 Total Return Index has displayed tremendous growth over the last 60 years – but is the market becoming more volatile?

- Certainly, the index level (left axis – blue line) post-1998 looks visually very different from pre-1998
- However, due to scale, the index chart can be very deceiving. If changes are roughly lognormal, then the log index (right axis – gold line) should look roughly linear:
  - Does it?
  - Most of the time, "yes." However, there are some outliers….
- This is what characterizes the real world:
  - Negative skew in returns
  - Volatility clustering
  - Bull and bear markets "outside the norm," indicating "fat tailed" returns
- The question arises: is the market really different post-1998 (say) versus pre-1998 and in particular, is post-2008 "qualitatively different" from prior markets?

S&P500 Total Return Index
31 Dec 1949 = 16.79

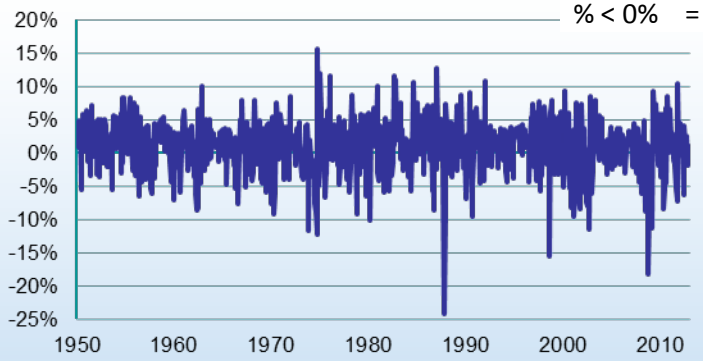


Looking at log returns and realized volatility levels by month, it isn't obvious that markets have become more variable

- The market continues to display certain characteristics already captured by many real world models – for example:
  - Negative skew in returns – top chart
  - Volatility clustering – bottom chart
- All real world models, no matter how complex, have parameters that control drift (trend/mean) and volatility (variance about the drift/mean)
- On a purely statistical basis, the historic data tell us a lot more about volatility than drift
- While volatility exhibits clustering and epochs of "highs" and "lows," it seems to be a rather well-behaved process if we make allowance for a couple of key dates (e.g., Oct 1987)
- Returns, on the other hand, are more unpredictable in the shorter term and hence justifiably the "drift" parameters for real world models should be set as much by pragmatism and prudence as by history

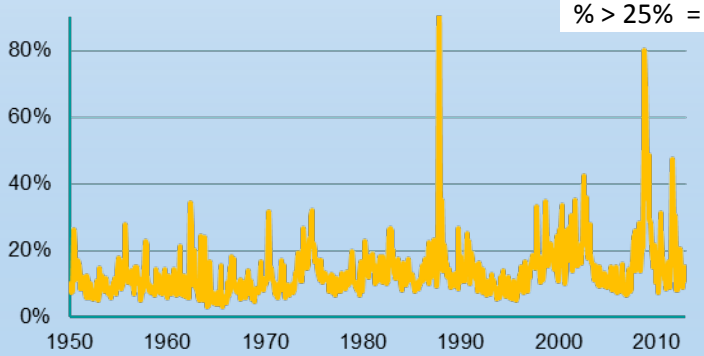
S&P500 Total Return Index, 1950-2012
Monthly Log Returns

Average = 0.87%
Median = 1.16%
% < 0% = 37.3%



S&P500 Total Return Index, 1950-2012
Monthly Volatility[1] (Annualized)

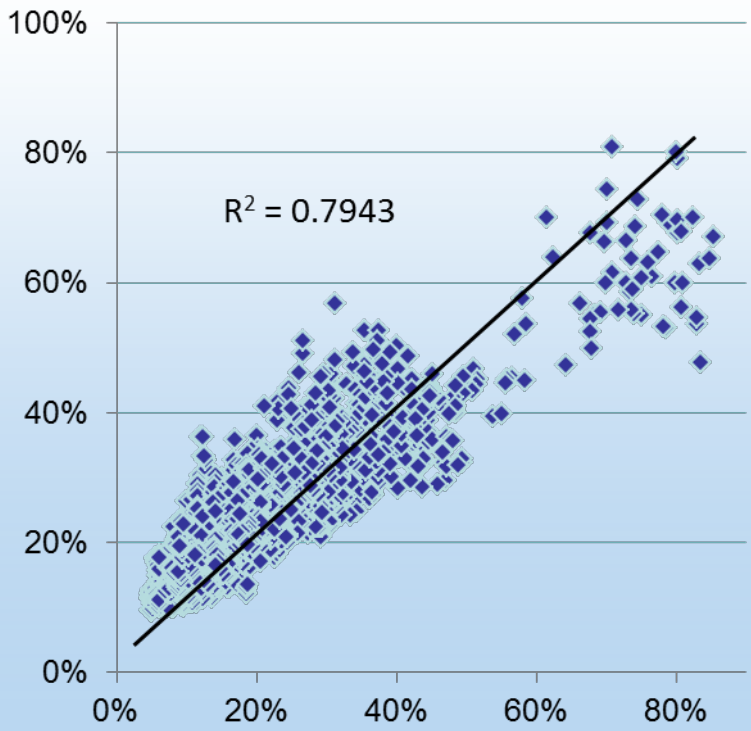
Average = 13.3%
Median = 11.3%
% > 25% = 6.75%



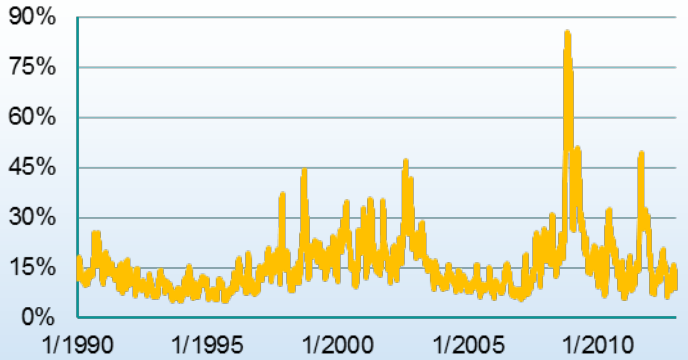
1. The realized volatility in a given month is calculated as the (annualized) standard deviation of daily log returns.

For completeness, we also examined the VIX[1] to see if it provides additional information regarding market volatility

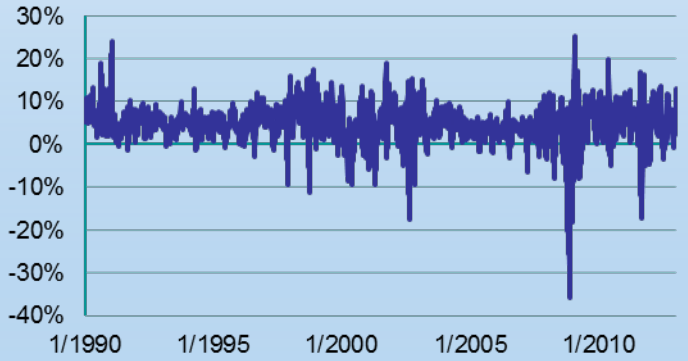
Daily VIX versus 21-day Rolling Realized[2] Volatility
1990.01.02 – 2012.12.31



Daily Rolling Realized[2] Volatility
1990.01.02 – 2012.12.31



Daily Difference[3] between VIX and Realized[2] Volatility
1990.01.02 – 2012.12.31



1. VIX is a trademarked ticker symbol for the Chicago Board Options Exchange Market Volatility Index, a popular measure of the implied volatility of S&P 500 index options.
2. Annualized standard deviation of S&P500 log total returns for the prior 21 days.
3. The chart shows the VIX minus realized volatility by day.

The VIX does not provide additional information; as such, we can use historic realized volatility estimates for our analysis and SLV parameterization

- It is not surprising to find a close relationship between the VIX and realized volatility[1] (R^2 = 0.7943)
- While the VIX is on average about 4.5% higher than realized volatility, it is not uniformly higher:
  - Furthermore, the VIX is much less variable
- From the previous slide, we see that the VIX is almost always lower than realized volatility when realized volatility is high (>35% annualized).
- The relationship of the VIX to real world returns does not suggest anything more than:
  - The VIX reacts to what is happening in the real world; and
  - The VIX includes a premium for the cost of hedging risk
- As such, for the purposes of parameterizing the SLV model, we conclude that it is preferable to infer return volatility characteristics from the real world index data than to use the VIX

Equity volatility characteristics 1990 – 2012

Statistic	Realized volatility	VIX
Minimum	4.86%	10.79%
2.5%	6.30%	11.30%
5.0%	7.15%	11.84%
10.0%	7.74%	12.34%
50.0%	13.96%	19.09%
90.0%	26.51%	29.38%
95.0%	33.16%	34.29%
97.5%	38.14%	38.02%
Maximum	79.95%	62.25%
Average	15.98%	20.45%
Stdev	9.52%	7.85%
% > 25%	12.68%	21.38%
% > 35%	3.62%	5.07%

1. Realized volatility is expressed as a rolling 21-day standard deviation of daily log S&P500 index returns.

However, given market experience post-2003 (when the original calibration criteria were developed), it is natural to ask:
Are the current equity return criteria still appropriate for real world valuation?

1

Is the market more volatile than previously?

- When observing the historic data, people often view the more recent past as "more relevant" – even though we are mostly concerned with long-term potential outcomes
- Volatility is unobservable and hence we need to rely on statistical analysis over a range of economic cycles, not single periods

2

Do we have lower expectations of returns?

- Perhaps. While the data still suggest long-term total growth of about 9-11% p.a., it is hard to accept this in the shorter/near term
- Still, most companies target 10-15% ROEs
- The existing criteria already reflect an 8.75% annualized long-run total return despite the data showing higher average returns

Two specific and inter-related questions arise regarding the existing NAIC criteria:

1. Would the criteria materially change after including post-2003 (i.e., up to Dec 2012) data, all else being equal?
2. Do the existing criteria contemplate the experience of 2008/2009 "at an appropriate level of likelihood"?

The “pre-packaged” scenarios derived from the SLV model readily satisfies the requirements; the SLV model is a good point of reference

- Notably, the Stochastic Log Volatility (SLV) model was parameterized from historic S&P500 data (1955-2003), but the drift was constrained so that the expected long-run average total return was 8.75% p.a.
  - This had the effect of shifting the left AND right tails to lower values for time horizons greater than 1 year
  - In short, it places more emphasis on the “put option” characteristics of the market versus the “call option” attributes
  - 1-year results are dominated more by volatility characteristics
- Many models can readily satisfy these criteria (i.e., suitably “fat-tails”):
  - Importantly, SLV and RSLN2[1]
  - Simpler models, like the normal distribution for log returns (the cornerstone of Black-Scholes-Merton option pricing), can readily satisfy the criteria with suitable adjustments

Existing (2005) Equity Return Calibration Criteria

Percentile	1 year	5 years	10 years	20 years
2.5%	0.78	0.72	0.79	
5.0%	0.84	0.81	0.94	1.51
10.0%	0.90	0.94	1.16	2.10
90.0%	1.28	2.17	3.63	9.02
95.0%	1.35	2.45	4.36	11.70
97.5%	1.42	2.72	5.12	

Stochastic Log Volatility Model (pre-packaged scenarios)

Percentile	1 year	5 years	10 years	20 years
2.5%	0.76	0.72	0.77	
5.0%	0.82	0.81	0.92	1.41
10.0%	0.89	0.93	1.12	1.83
90.0%	1.30	2.22	3.81	10.15
95.0%	1.37	2.48	4.44	12.93
97.5%	1.44	2.73	5.17	

1. Regime switching lognormal model for index prices. See Mary Hardy, NAAJ.

Given recent (post-2003) data, how would we approach the following issues?

1

Is the market more volatile than previously?

- As with previous Academy work, perform volatility analysis on the daily S&P500 total returns
- Assess intra-month and long-run volatility
- Examine clustering, mean-reversion, etc.
- Re-parameterize the SLV model

2

Do we have lower expectations of returns?

- Full analysis is out of scope
- Drift estimates are highly uncertain
- There is no evidence that expected long-run total returns (with dividends) will be less than 8.75%
- As such, it seems reasonable to maintain the current constraints “built into” the criteria

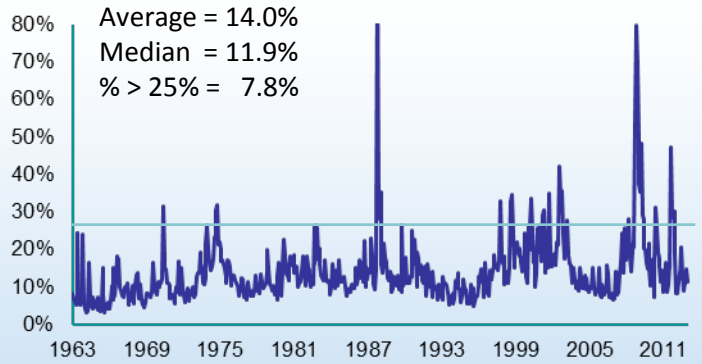
As such, the current analysis will consider the following questions:

- How would the calibration criteria change, after incorporating post-2003 data, all else being equal?
- At what confidence level and horizon did the existing criteria “anticipate” the recent (2008-2009) financial crisis?

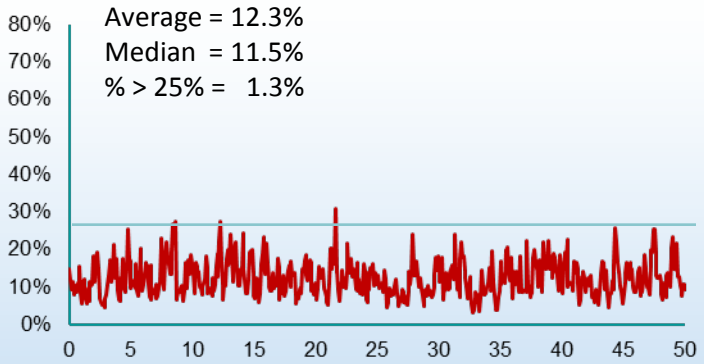
2 Analysis of S&P500 Equity Returns – Including the Financial Crisis

The original SLV model[2] produces "volatility paths" consistent with history – relatively fast volatility mean-reversion, clustering and occasional "spikes"

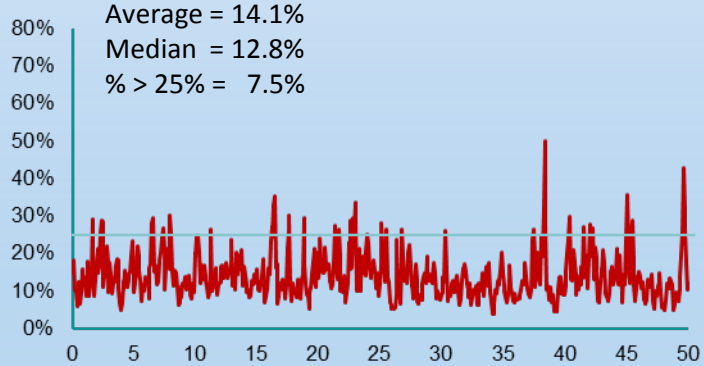
Historic realized S&P500 volatility[1] (annualized)
1962 – 2012 (50 years)



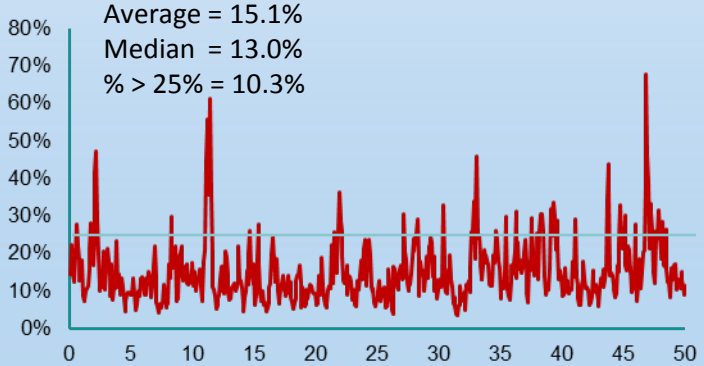
SLV Original – Sample[2] process volatility
Least volatile scenario (#9008, s=4.60%)



SLV Original – Sample[2] process volatility
Average volatility scenario (#7269, s=6.10%)



SLV Original – Sample[2] process volatility
Most volatile scenario (#3485, s=8.23%)



1. The realized volatility in a given month is calculated as the (annualized) standard deviation of daily log returns.
2. This is an extended version of the "2005 pre-packaged scenarios" (same random numbers) based on the original 2005 parameters. Starting volatility = 14.76% (same as pre-packaged).

We updated the SLV parameters by incorporating market return data up to the end of 2012 (an additional nine years vs. the original model)…

v1 **Original[1] (2005) parameterization[2]**
S&P 500 returns, 1956 – 2003 (48 years)

τ	12.515%
ϕ	0.35229
σ	0.32645

v2 **Updated MLE parameterization[2]**
S&P 500 returns, 1956 – 2012 (57 years)

τ	12.040%
ϕ	0.30142
σ	0.33741

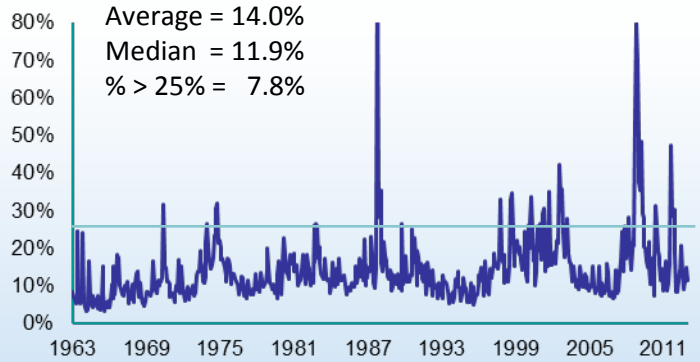
Interpretation

v1 vs. v2	• Reflects impact of including more recent market data on the SLV parameters (all else being equal) – lower target volatility, but lower mean-reversion strength and marginally higher process variation • A priori, it is difficult to know whether v2 would produce uniformly "more volatile paths" than v1; hence, we need to simulate scenarios and conduct some statistical analysis…

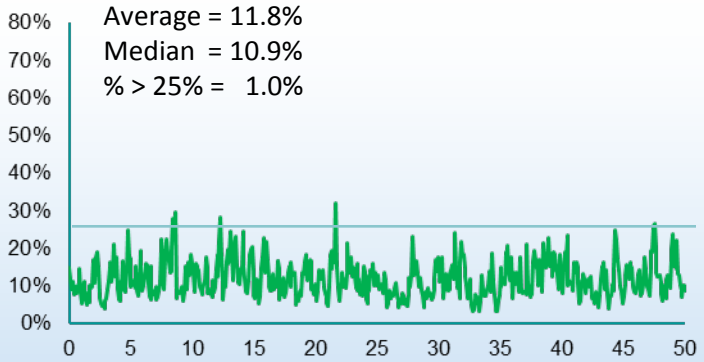
1. See Report entitled *C3 Phase II Risk-Based Capital for Variable Annuities: Pre-Packaged Scenarios*, Academy LCAS March 2005.
2. τ = target process annualized volatility (not stock return volatility); ϕ = mean-reversion strength (monthly); σ = standard deviation of log volatility process.

The updated SLV model[2] produces volatility paths that are almost indistinguishable from the original 2005 parameterization (compare to p.15)

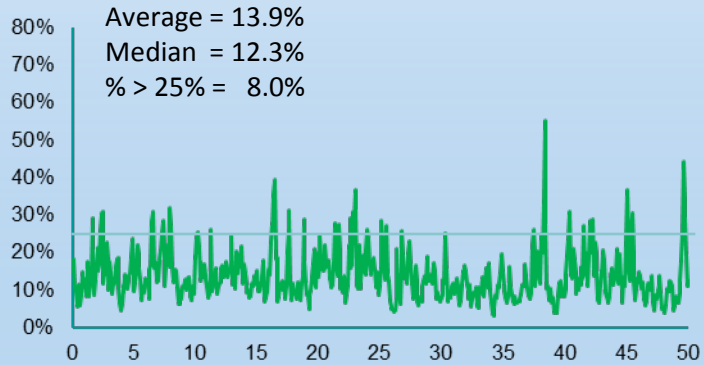
Historic realized S&P500 volatility[1] (annualized)
1962 – 2012 (50 years)



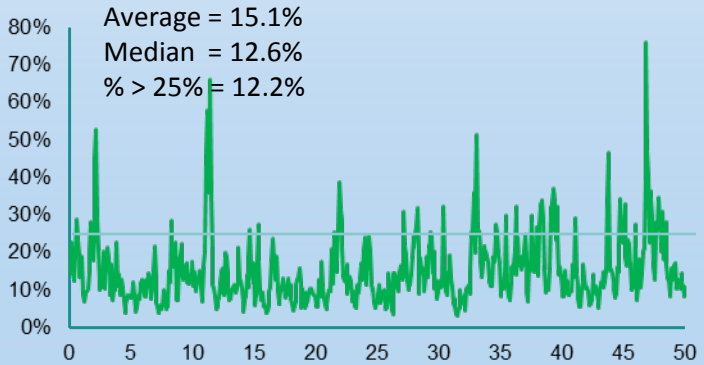
SLV Updated – Sample[3] process volatility
Least volatile scenario (#9008, s=4.76%)



SLV Updated – Sample[3] process volatility
Average volatility scenario (#7269, s=6.59%)



SLV Updated – Sample[3] process volatility
Most volatile scenario (#3485, s=9.05%)



1. The realized volatility in a given month is calculated as the (annualized) standard deviation of daily log returns.
2. Updated MLE parameters for the stochastic log volatility process based on daily S&P500 total return data, 1955.12.31 to 2012.12.31 inclusive.
3. These volatility simulations use the same random numbers as those underlying the charts displayed on page 15 (see previously). Starting volatility = 14.76% (same as pre-packaged).

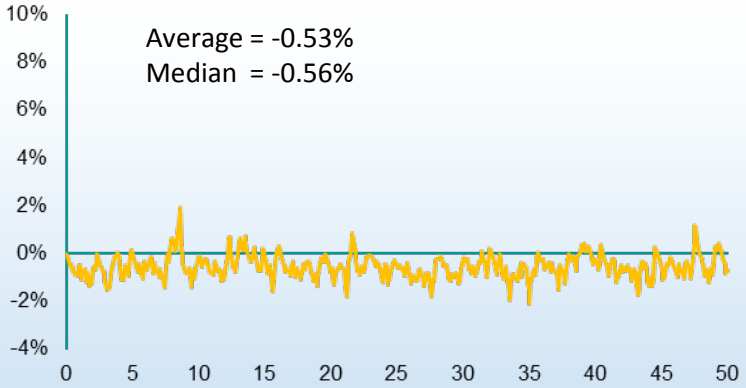
All else being equal, updated (v2 parameters) volatility paths are modestly more diverse (extreme) – at the lower and upper percentiles

Volatility characteristics

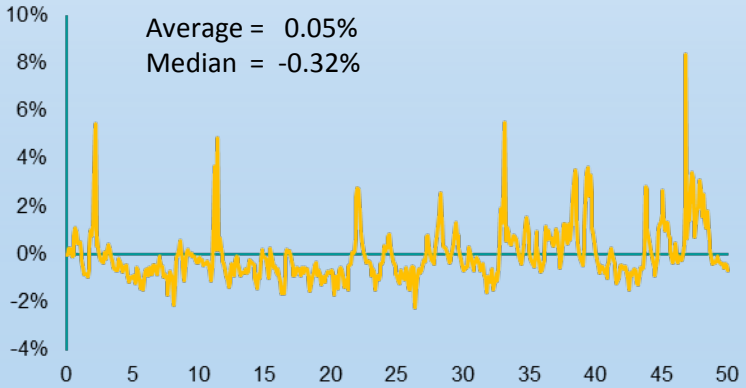
Statistic[3]	SLV[1] v1 Original 2005 (1956-2003)	SLV[1] v2 Updated 2012 (1956-2012)	Historic Values[2] (1956-2012)
2.5%	5.41%	4.80%	5.19%
5.0%	6.19%	5.56%	5.90%
10.0%	7.23%	6.59%	6.92%
50.0%	12.51%	12.04%	11.52%
90.0%	21.66%	22.00%	21.81%
95.0%	25.31%	26.09%	27.55%
97.5%	28.97%	30.21%	33.02%
Average	13.72%	13.44%	13.59%
Stdev	6.15%	6.66%	8.20%

- Overall characteristics (v1 vs. v2) are not very different; to understand the impact on equity returns, we need to examine gross wealth ratios (see the following pages)
- See right panel for some insight into path-specific differences

SLV sample volatility process differences (Updated – Original)
Least volatile scenario (#9008)



SLV sample volatility process differences (Updated – Original)
Most volatile scenario (#3485)



1. Annualized process volatility from the monthly SLV model. The actual volatility of returns is higher (see footnote 3).
2. The historic realized volatility in a given month is calculated as the (annualized) standard deviation of daily log returns.
3. Based on analysis using a daily version of the SLV, the average monthly volatility of simulated returns is roughly 0.70% higher than the SLV process volatility.

The updated SLV parameterization (compared to original) implies only very minor changes to the existing equity return calibration criteria

SLV – GWRs – Original 2005 (1955-2003) Parameters

Percentile	1 year	5 years	10 years	20 years
2.5%	0.756	0.722	0.771	
5.0%	0.818	0.807	0.923	1.411
10.0%	0.886	0.933	1.124	1.832
50.0%	1.089	1.452	2.089	4.274
90.0%	1.297	2.222	3.805	10.153
95.0%	1.370	2.481	4.441	12.926
97.5%	1.437	2.731	5.173	

Existing (2005) Equity Return Calibration Criteria

Percentile	1 year	5 years	10 years	20 years
2.5%	0.78	0.72	0.79	
5.0%	0.84	0.81	0.94	1.51
10.0%	0.90	0.94	1.16	2.10
90.0%	1.28	2.17	3.63	9.02
95.0%	1.35	2.45	4.36	11.70
97.5%	1.42	2.72	5.12	

SLV – GWRs – Updated (1955-2012) Parameters

Percentile	1 year	5 years	10 years	20 years
2.5%	0.752	0.705	0.758	
5.0%	0.814	0.804	0.912	1.392
10.0%	0.886	0.930	1.112	1.803
50.0%	1.089	1.450	2.080	4.231
90.0%	1.296	2.202	3.767	10.036
95.0%	1.370	2.467	4.400	12.766
97.5%	1.437	2.710	5.114	

Implied (Updated) Calibration Criteria

Percentile	1 year	5 years	10 years	20 years
2.5%	0.78	0.70	0.78	
5.0%	0.84	0.81	0.93	1.49
10.0%	0.90	0.94	1.15	2.07
90.0%	1.28	2.15	3.59	8.90
95.0%	1.35	2.44	4.32	11.54
97.5%	1.42	2.70	5.06	

The SLV model fits the historic monthly data extremely well even when parameterized to different periods (e.g., 1956-2003 and 1956-2012)

Statistics for monthly log total returns

Statistic	Historic 1956-2003	SLV Original		Historic 1956-2012	SLV Updated
0.01%	n/a	-0.3363		n/a	-0.3609
0.17%	-0.2426	-0.1953		-0.2426	-0.2121
2.5%	-0.0863	-0.0910		-0.0870	-0.0912
5.0%	-0.0643	-0.0673		-0.0660	-0.0666
10.0%	-0.0448	-0.0448		-0.0465	-0.0435
50.0%	0.0116	0.0087		0.0116	0.0086
90.0%	0.0560	0.0541		0.0552	0.0530
95.0%	0.0719	0.0707		0.0719	0.0700
97.5%	0.0863	0.0872		0.0857	0.0873
99.83%	0.1560	0.1546		0.1560	0.1652
99.99%	n/a	0.2359		n/a	0.2496
Average	0.0083	0.0061		0.0077	0.0060
Stdev	0.0427	0.0436		0.0429	0.0436
Skew	-0.59	-0.67		-0.66	-0.74
Kurt	2.42	3.97		2.46	5.06
% < 0%	38.7%	39.7%		38.3%	39.3%

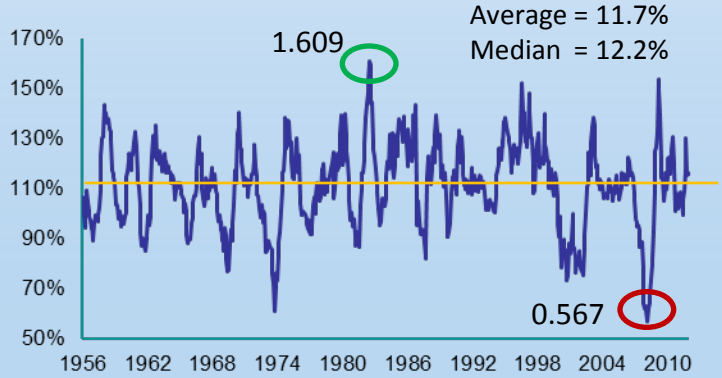
- The "centers" (mean and median) of the SLV distributions are lower than history by design – the calibration criteria require an average return no higher than 8.75% (annualized)
- The SLV model produces similar skewness to history (relative density to the left/right of the mean)
- Similar to history, roughly 39% of returns are negative
- However, the SLV model exhibits much fatter tails (higher kurtosis):
  - The worst 1-month historic return is -24.3% (October 1987 – approx. a 0.17% event), but the SLV can produce lower returns (see chart)
- Lowest 1-month return post-2003 is -18.3% (October 2008), the 2nd worst return in history[1]
  - Ex post (i.e., based on monthly data from 1956 – 2012), this was a 0.29% event[2]
  - The original SLV model (fit to 1956–2003 only) exhibits a monthly log return of -17.3% at this confidence level and hence "anticipates" such adversity with quite reasonable frequency

1. In this context, "history" means post 1955 using S&P500 index total return data.
2. The monthly data 1956-2012 include 684 monthly returns. Hence, the 2nd worst return is an estimate of the 2/685 = 0.29% confidence level.

While the SLV model fits the monthly data extremely well, it is also important to look at annual returns…

- As we have seen, the historic data convey much information about volatility (e.g., clustering, reversion to a target level, etc.) and hence insight into short-term (daily and monthly) equity returns
- Unfortunately, the historic data provide relatively little direct, statistically credible insight into long-term (>5 years) returns, but we can use history to evaluate the calibration criteria and SLV model over 1-year
- First, we examine the historic rolling[1] 1-year returns to see if there is some pattern in the returns following a large drop in the index:

Historic monthly rolling PTP[1] annual accumulation
1956.12 – 2012.12 (56 years)



- Annual PTP returns looks mean-reverting, with strong "recovery returns" after large drops:

	PTP Return	12m Ending	Subsequent[2]
Worst	-43.3%	2009.02	+53.6%
Best	+60.9%	1983.06	-4.5%

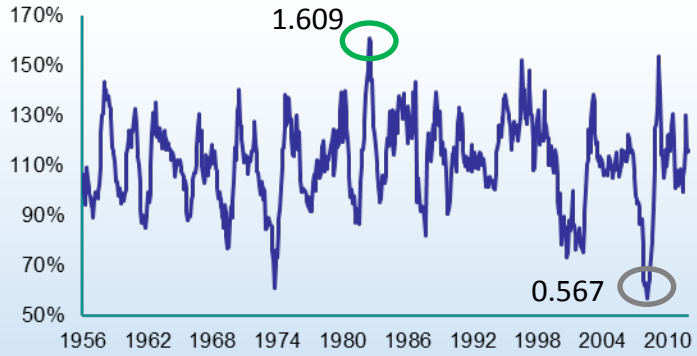
- Average (median) 1y "recovery return" after the index has fallen >15% over 1-year is +19.7% (+25.3%)
- While this is not clear evidence of "mean-reversion" in returns (or index levels) longer-term, it does suggest we need to take care in analysing annual returns

Historic daily S&P500 Total Return Index
2003.12.31 – 2012.12.31 (9 years)

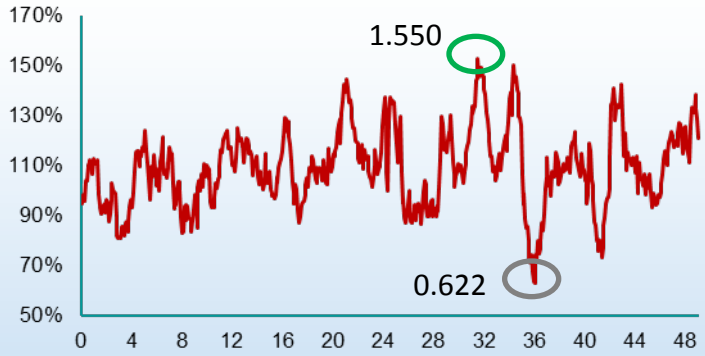
1. Point-to-point (rolling) accumulation factors based on month-end S&P500 Total Return index values relative to the index level 1-year prior.
2. These are the returns for the 12 months following the date specified (e.g., +53.6% from 2009.02 to 2010.02).

The SLV model[2] does a good job of capturing the characteristics of – and potential variation in – annual PTP[1] returns relative to history

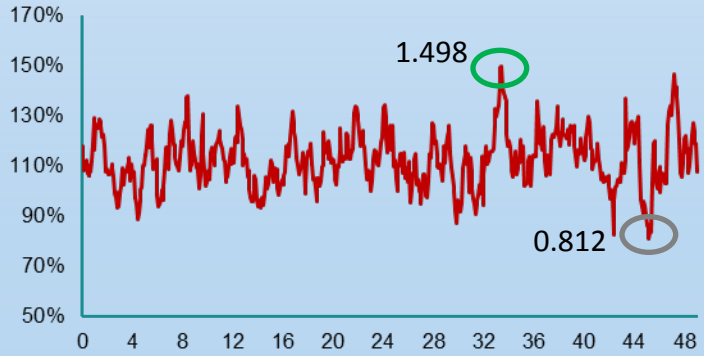
Historic monthly rolling PTP[1] annual accumulation
1956.12 – 2012.12 (56 years), Avg=11.7%, Med=12.2%



SLV Original – monthly rolling PTP[1] annual accumulation
Average PTP volatility (#1762), Avg=9.0%, Med=8.6%



SLV Original – monthly rolling PTP[1] annual accumulation
Lowest PTP volatility (#5492), Avg=12.5%, Med=11.9%

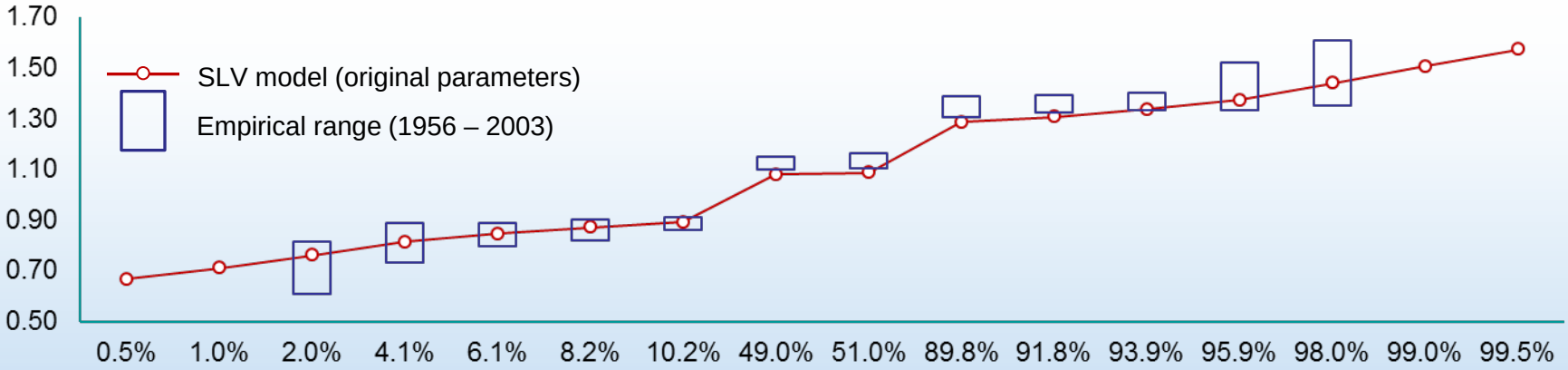


SLV Original – monthly rolling PTP[1] annual accumulation
Highest PTP volatility (#4438), Avg=8.9%, Med=8.9%

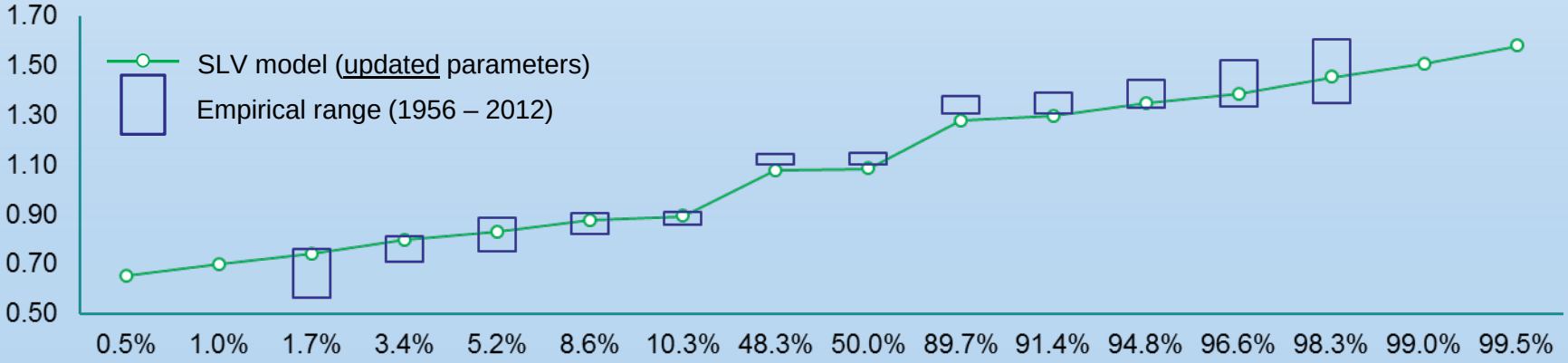
1. Point-to-point (rolling) accumulation factors based on month-end S&P500 Total Return index values (actual or simulated) relative to the index level 1-year prior.
2. Note: the SLV model with updated parameters (based on 1956–2012 data) produces almost identical values.

The SLV statistics for 1-year gross wealth ratios lie within[1] the historic empirical ranges, especially in the critical tails of the distribution

1-Year Gross Wealth Ratios for S&P500 Total Return Index
Original SLV model vs. historic empirical ranges (1956-2003)



1-Year Gross Wealth Ratios for S&P500 Total Return Index
Updated SLV model vs. empirical ranges (1956-2012)

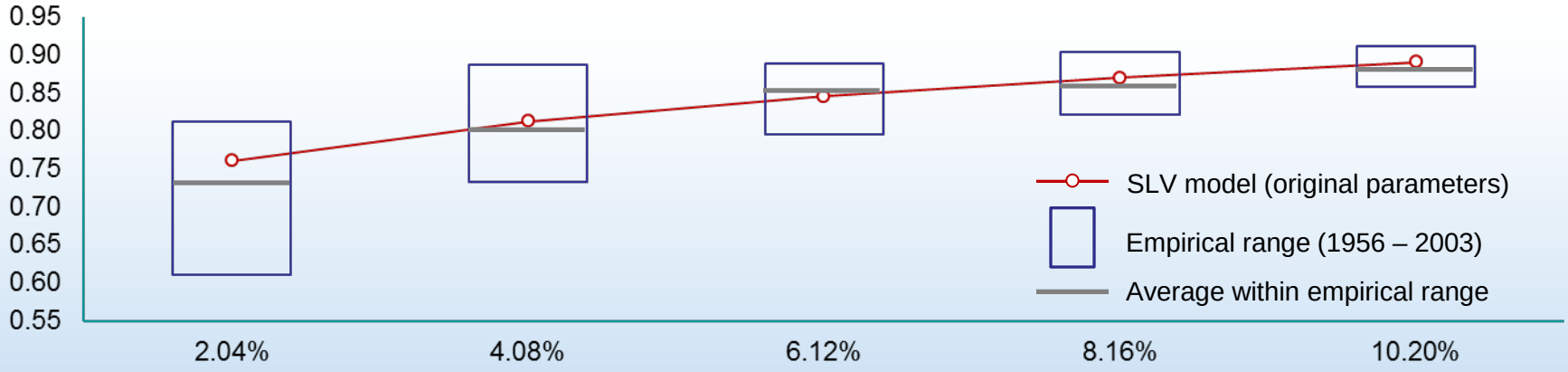


1. The SLV values near the median do not lie within the observed historic ranges because the expected return for the SLV model is constrained not to exceed 8.75% per annum.

Focusing on the left-tail, we see that the SLV statistics are reasonably close to the averages within the empirical ranges – evidence of good fit

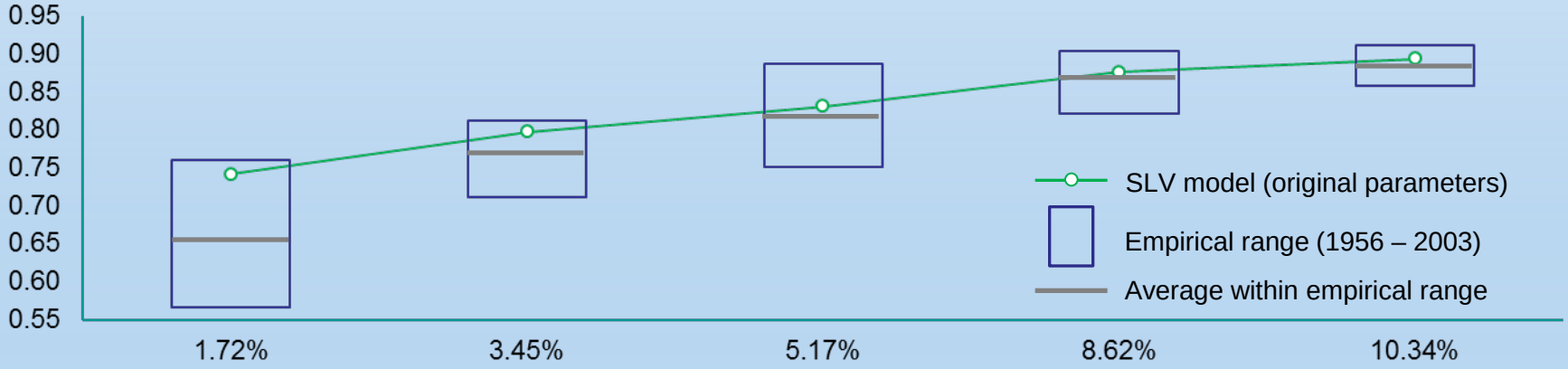
1-Year Gross Wealth Ratios for S&P500 Total Return Index

Original SLV model vs. historic empirical ranges (1956-2003)



1-Year Gross Wealth Ratios for S&P500 Total Return Index

Original SLV model vs. historic empirical ranges (1956-2012)



The existing[1] 1-year calibration points are still within the empirical ranges suggested by the historic data, including post-crisis experience

- The 1955.12.31 – 2003.12.31 monthly S&P500 TR data series provides 576 non-overlapping end-of-month returns and 48 non-overlapping observations of annual returns (i.e., for each calendar year)
- Since we can choose different starting points for non-overlapping returns, there are 12 different annual series (i.e., Jan-Jan, …, Dec-Dec). The series are not independent, but provide slightly different empirical estimates of the underlying distributions
- Similarly, the 1955.12.31 – 2012.12.31 monthly data provide 684 non-overlapping EOM returns and 57 non-overlapping observations of annual returns
- The table at right provides estimates of 1-year return statistics based on the empirical observations[2,3]
- The historic data are too sparse (e.g., ranges too wide) to provide credible empirical calibration points; as such, we need to "extrapolate" from these data based on fitted models (see 2005 Academy LCAS report)
- More recent data (post-2003) have shifted the deep left-tail, but they do not suggest a major overhaul

Empirical statistics for non-overlapping S&P500 Total Return Index 1-year Gross Wealth Ratios (low, *average*, high)

Stat	Calib. Point[1,3]	1956-2003	1956-2012
# obs		48	57
1.72%		-	(0.567, *0.653*, 0.760)
2.04%		(0.610, *0.745*, 0.813)	-
2.50%	**0.78**	(0.638, *0.759*, 0.830)	(0.632, *0.701*, 0.784)
3.45%		-	(0.712, *0.759*, 0.813)
4.08%		(0.734, *0.806*, 0.889)	-
5.00%	**0.84**	(0.761, *0.826*, 0.889)	(0.747, *0.812*, 0.881)
5.17%		-	(0.751, *0.818*, 0.889)
6.12%		(0.795, *0.850*, 0.890)	-
6.90%		-	(0.795, *0.850*, 0.890)
8.16%		(0.822, *0.868*, 0.905)	-
8.62%		-	(0.822, *0.868*, 0.905)
10.00%	**0.90**	(0.855, *0.885*, 0.912)	(0.852, *0.883*, 0.911)
10.20%		(0.859, *0.887*, 0.913)	-
10.34%		-	(0.859, *0.886*, 0.913)

1. See Table 1 in Appendix 2 of the June 2005 LCAS Report for the existing equity return calibration criteria, also shown on slides 3, 12 and 19.
2. For example, the historic 1956-2003 month end data provide 12 series of annual returns with 48 observations each. The lowest return in each series is an estimate of the 1/49th percentile (2.04%). As such, we obtain a range for the 2.04% value (i.e., lowest and highest observations across the 12 series at that confidence level).
3. Ranges in the shaded regions at the calibration criteria percentiles (i.e., 2.5%, 5% & 10%) are obtained by linear interpolation from the empirical estimates at neighboring points.